

# Theoretical and Experimental Study of Wave Propagation in Stressed Plate

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## Abstract

Detection of stresses in structures is widely required. Acoustic stress measurement is a Non-Destructive Evaluation (NDE) method. In this paper wave propagation in a stressed plate is studied, using numerical simulations. We used ABAQUS, a commercial finite element software, to model the plate. Many parameters affect the wave propagation as plate thickness, plate length, and frequency of the wave signal are studied. When compressive and tensile loads are applied to a plate, the wave propagation is investigated. It was concluded that the effect of stress on the wave propagation decreases with increasing plate length and also with increasing plate thickness, wherewith increasing frequency the effect of stress in wave propagation is clearer.

**Keywords:** Stress, Acoustic waves, Lamb wave, Acoustoelasticity.

*Tob Regul Sci.*™ 2021;7(5): 2275-2285

DOI: [doi.org/10.18001/TRS.7.5.140](https://doi.org/10.18001/TRS.7.5.140)

## 1 INTRODUCTION

In civil engineering one of the main purposes for the material is to support loads. Every service structure element is subjected to stress. When one tries to detect the stress level in a certain structure element, one of the suggested methods for measuring stress is to remove a piece of material from the element and introduce a load cell to measure force, but this method is destructive [1]. Recently there are several non-destructive techniques (NDT) for the detection of stress. These techniques such as laser scattering, magnetic Barkhausen noise, X-ray diffraction, pulse thermography, atomic force microscopy, and scanning acoustic microscopy are also being used for the investigation of material surfaces. The selection of a suitable

technique is crucial for the accuracy of measurement. In some cases, it may be necessary to use more than one technique [2]. Ultrasonic waves have been widely used for NDE and structural health monitoring (SHM) in civil, aeronautical, electrical, and mechanical applications for many decades [3-4]. Lamb waves are gaining popularity in NDEs due to their capacity to travel over great distances and penetrate hidden areas with little energy loss [3]. The acoustoelastic effect, defined as the stress dependence of ultrasonic wave velocity, is fully represented when the higher-order strain components of the material's stress-strain relationship are incorporated in the constitutive equations [5-6]. A bulk wave is one that propagates indefinitely across a medium. Bulk waves

are divided into two kinds according to their propagation and direction of vibration. When the vibration and propagation directions are parallel, a longitudinal wave is generated; when they are perpendicular, a shear wave is formed. On the other hand, a directed wave occurs when a wave propagates along the medium's or two media's boundaries. Guided waves are widely used in the Nondestructive Testing (NDT) industry due to their high sensitivity to characteristics and extended propagation distance. Lamb waves are a kind of guided wave that propagate through plates [11]. N. Gandhi and colleagues [12] developed an acoustoelastic Lamb wave propagation theory for an isotropic medium subjected to a biaxial homogeneous stress field. M. Mosbuth and colleagues [13] examined the effect of uniaxial stress on higher-order lamb wave mode propagation. This research examines the wave propagation in strained plates when subjected to uniaxial stress. The effect of different variables such as plate thickness, plate length, and the frequency of the wave signal is studied.

## 2 ACOUSTOELASTIC EFFECT

The propagation speed of an ultrasonic wave in elastic material is affected by the presence of stress. As a result, stress may be measured by measuring the velocity of ultrasonic wave propagation [7]. The speed of sound in a material may be estimated using the second and third-order elastic constants. The wave speed of a longitudinal acoustic wave moving parallel to the tension in a material is given by:

$$\rho_0 c_{Lx}^2 = \lambda + 2\mu - \frac{\sigma}{3K_0} \left[ 2L + \lambda + \frac{\lambda + \mu}{\mu} (4m + 4\lambda + 10\mu) \right]$$

(1)

where,  $\rho_0$  is the initial density,  $c_{Lx}$  is the longitudinal sound speed parallel to the applied stress  $\sigma$ ,  $\lambda$  and  $\mu$  are the Lamé or second-order elastic constants,  $K_0$  is the initial bulk modulus and  $L$  and  $m$  are

Murnaghan's or third-order elastic constants.

When the longitudinal wave is traveling perpendicular to the stress wave speed equation becomes:

$$\rho_0 c_{Ly}^2 = \lambda + 2\mu - \frac{\sigma}{3K_0} \left[ 2L + \lambda + \frac{2\lambda}{\mu} (m + \lambda + 2\mu) \right]$$

(2)

where,  $c_{Ly}$  is the longitudinal speed of sound perpendicular to the stress. For more simplification acoustoelastic constant  $K$  is defined. For each type of wave and direction of propagation relative to the stress, a different  $K$  must be given. Thus the ultrasonic speed in the material can be found as

$$K \sigma = \frac{\Delta c}{c}$$

(3)

where  $\sigma$  is the stress,  $c$  is the speed of sound of the unstressed material, and  $\Delta c$  is the speed of sound change. Acoustical constants may be established by subjecting materials to a range of loads and measuring the sound speed in those materials [8]. Z. Abbasi and D. Ozevin [9] used acoustoelasticity theory to explore the impact of ultrasonic frequency on measuring shear stress. This study investigates the effect of sensor frequency on stress measurement. K.F. Bompan and V.G. Haach [10] explored the use of ultrasonography for acoustoelasticity-based stress measurement in concrete prisms.

## 3 VERIFICATION WITH EXPERIMENTAL STUDY

The accuracy and the validity of the proposed model of the wave propagation in the steel plate are evaluating using the previous experimental work done by [QIU, et al (2019)]. ABAQUS program is used in this verification. Two steps are to propagation of simulate lamb wave in stresses aluminum plate. The dimensions of aluminum plate are 400mm in length, 200mm in width, and 2mm in thickness.

The plate is free in one end and in the fixed other. The load levels in the static step from 0MPa to 100MPa with increment 10MPa. Hanning windows are used as the wave excitation signal with a 200 kHz frequency. Fig.1 shows the experimental system. The material properties of the aluminum are 2700 density, 70GPa Young's modulus, and 0.33 Poissons ratio.

Fig.3 shows the Acceleration response of the wave propagation. By evaluating this figure by Fig.2 it is shown that there is no clear variation between the two figures.



Figure 1 Expermenat system [QIU, etal (2019)].

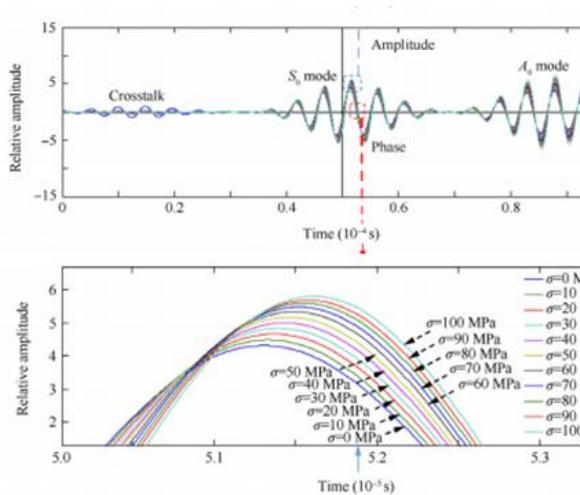


Figure 2 Experimental results [QIU, et al (2019)].

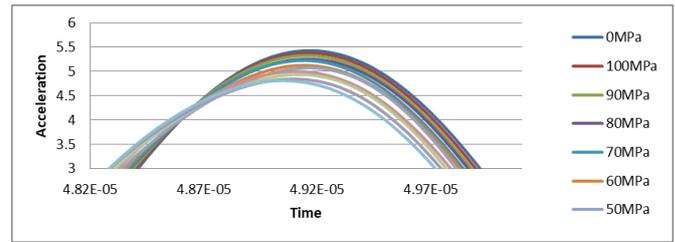
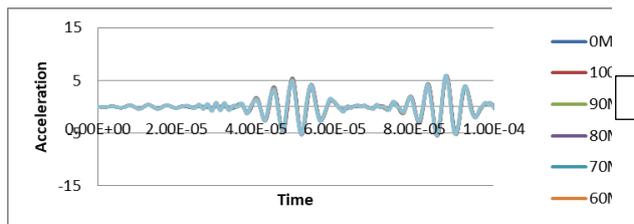


Figure 3 Numerical model results.

#### 4 ABAQUS MODEL

The effect of different variables on wave propagation in a stressed steel plate is investigated using the 3D ABAQUS model. The plate is secured at one end and loaded at the other, as shown in Fig. 4. The steel plate's material properties include a density of 7850 Kg/m<sup>3</sup>, a modulus of elasticity of 2\*10<sup>11</sup>N/m<sup>2</sup>, and a Poisson's ratio of 0.3. Two phases of wave propagation are represented in the strained plate. The first phase is stationary, during which the plate's compressive or tensile stresses are modeled, and the findings of this step serve as the starting condition for the second phase. The second step involves modeling wave propagation in a plate exposed to compressive or tensile stresses using ABAQUS explicit dynamic. At the transmitter site, the wave signal is applied, and the resultant wave is received at the reception site. In all cases, the transmitter point is 100 mm from the plate's fixed end and the receiver point is 100 mm from the plate's free end. The plate was constructed from solid cubic particles, and the homogenized model was meshed using an eight-node brick element. The wavelength determines the element mesh size, which is set to 1mm with a time step of 10<sup>(-7)</sup>sec.

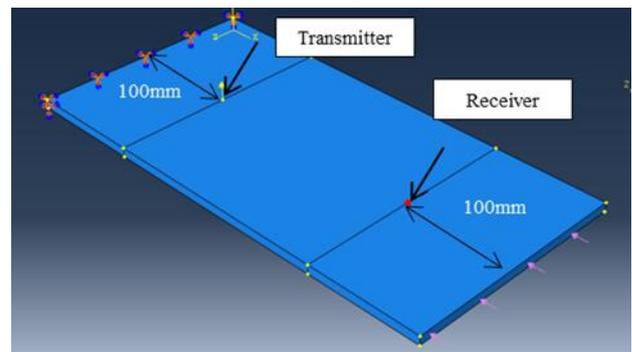
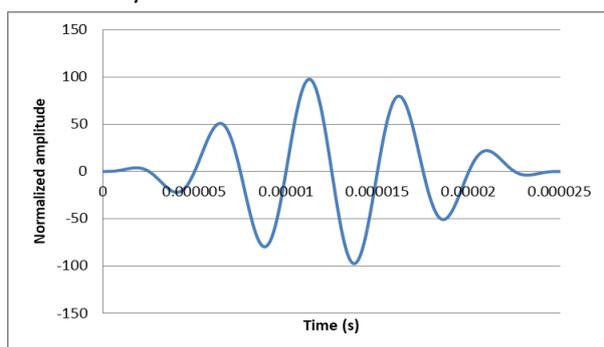


Figure 4 ABAQUS modeling of steel plate.

Hanning-window with five cycles and frequency of 200 kHz is used as an excitation signal into ABAQUS software as a forcing function to generate lamb wave as expressed in equation (3). Fig. 5 shows the Hanning signal used in ABAQUS.

$$E = A [1 - \cos(2\pi ft / N)] \sin(2\pi ft) \quad (4)$$

Where A is taken 50, f is the frequency 200 kHz, N is the number of cycles N=5, and  $t < N/f$ .



**Figure 5** Hanning window excitation signal.

### 5 EFFECT OF PLATE THICKNESS

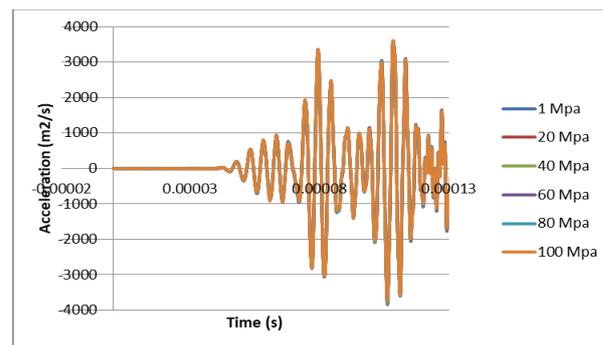
Four plates with different thicknesses are used to study the effect of variation in plate thickness in wave propagation. Every plate is studied under the effect of different levels of compressive or tensile stresses. These stresses are 1MPa, 20MPa, 40MPa, 60MPa, 80MPa, and 100MPa. Table (1) shows the dimensions of the plate.

**Table 1** Plates with a different thickness dimension

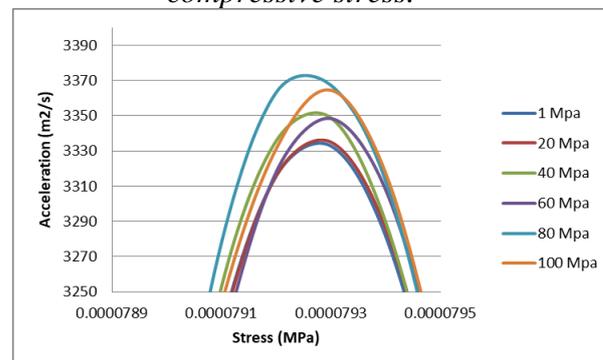
	Length (mm)	Width (mm)	Thickness (mm)
First plate	400	200	2
Second plate	400	200	4
Third plate	400	200	6
Fourth plate	400	200	8

Fig. 6 shows the variation of the received acceleration concerning the time at

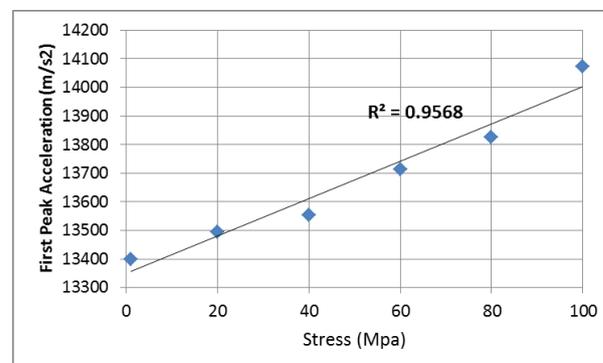
different levels of compressive stresses for the fourth plate. It is shown that there are no variations in time of flight but by zoom this figure at a marked point as in fig. 7, it is found that there is a slight change in the amplitude of the peak acceleration. Figures 8, 9, 10, and 11 show the relation between first peak acceleration and the applied stresses when these plates are under varied compressive stresses.



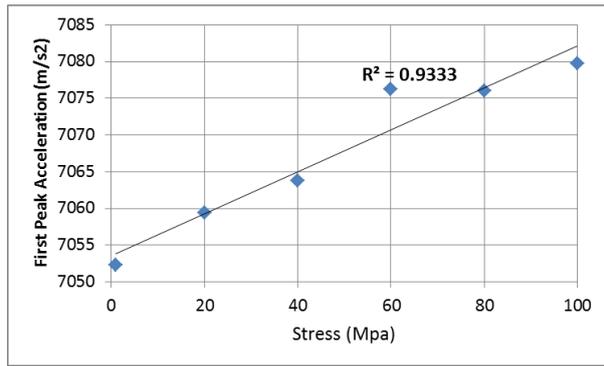
**Figure 6** Acceleration response at the received point of the fourth plate in case of compressive stress.



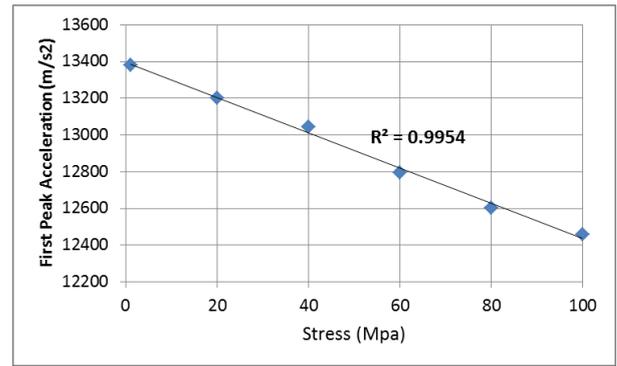
**Figure 7** Zoomed peak acceleration of the first plate in case of compressive stresses.



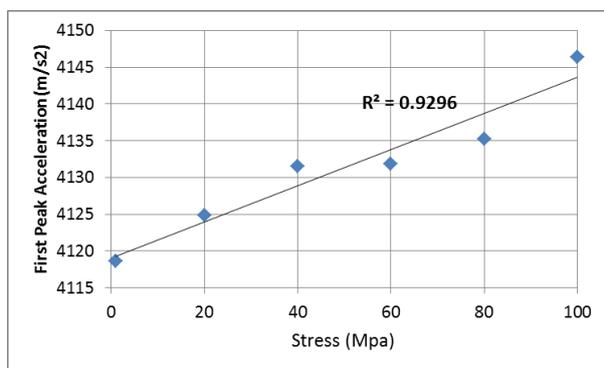
**Figure 8** Variation of peak acceleration of the first plate under compressive stresses.



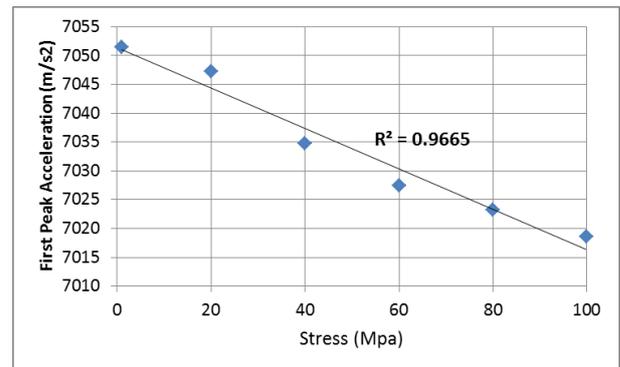
**Figure 9** Variation of peak acceleration of the second plate under compressive stresses.



**Figure 12** Variation of peak acceleration of the first plate under tensile stresses.

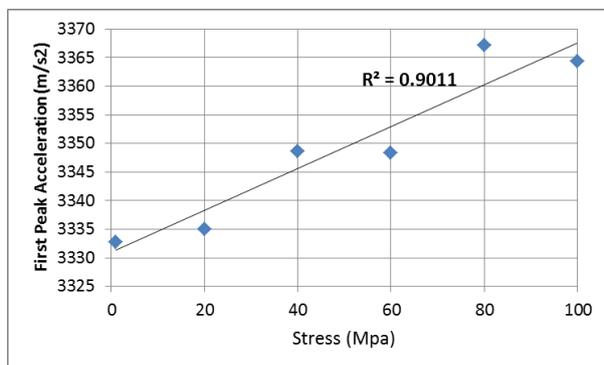


**Figure 10** Variation of peak acceleration of the third plate under compressive stresses.

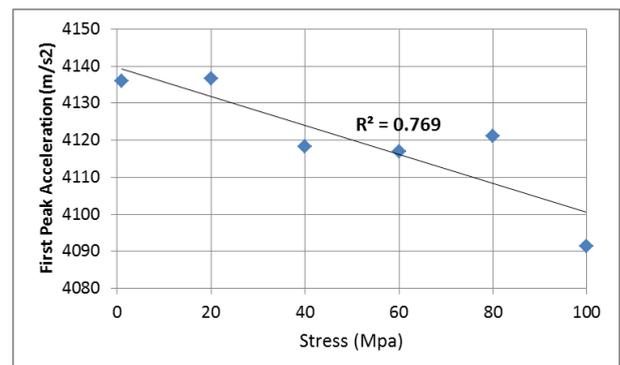


**Figure 13** Variation of peak acceleration of the second plate under tensile stresses.

It is shown that the peak acceleration of the received wave is slightly decreased with increasing tensile stresses.



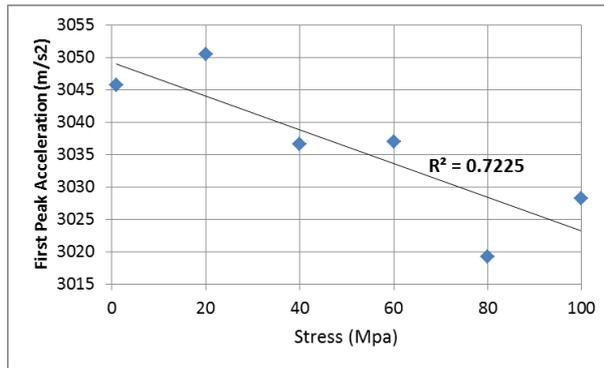
**Figure 11** Variation of peak acceleration of the fourth plate under compressive stresses.



**Figure 14** Variation of peak acceleration of the third plate under tensile stresses.

It is shown that the peak acceleration of the received wave is slightly increased with increasing compressive stresses.

Figures 12, 13, 14, and 15 show the relation between first peak acceleration and the applied stresses when these plates are under varied tensile stresses.

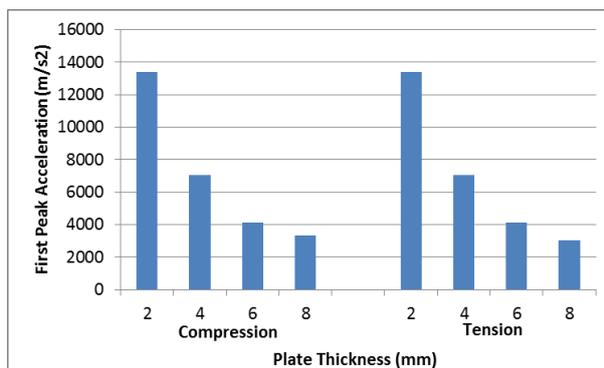


**Figure 15** Variation of peak acceleration of the fourth plate under tensile stresses.

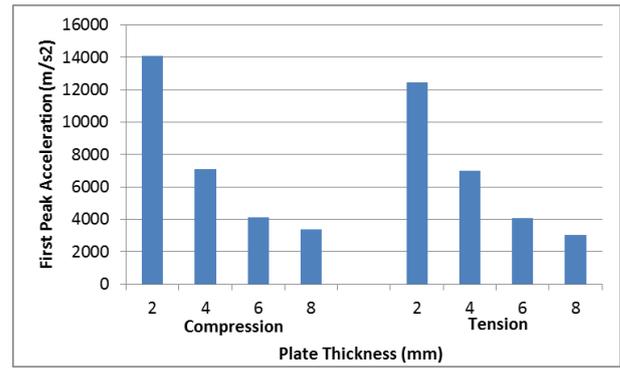
It is noted that for the plates under compressive or tensile stresses, the regression factor is decreased with the increase of the plate thickness because the small plate thickness leads to less distortion of the guided wave.

Fig. 16 shows the variation of the first peak acceleration for the four plates when subjected to compressive or tensile stresses 1 Mpa. It is found that there is a sharp decrease in the peak acceleration when the plate thickness is increased, and there is no change between the peak acceleration in all plates when subjected to compressive or tensile stresses.

Fig. 17 shows the variation of the first peak acceleration for the four plates when subjected to compressive or tensile stresses of 100 Mpa.



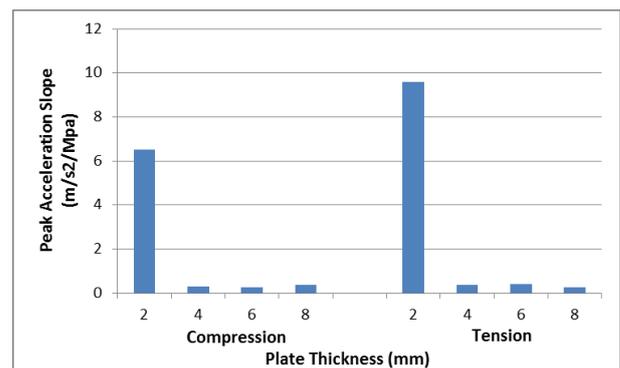
**Figure 16** Variation of peak acceleration of plates with different thicknesses under compressive or tensile stress (1 Mpa).



**Figure 17** Variation of peak acceleration of plates with different thicknesses under compressive or tensile stress (100 Mpa).

It is found that there is a slight increase in the peak acceleration of all plates when subjected to compressive stress than tensile stresses because the particles of the plate are converged when subjected to compressive stress, while the particles have diverged when subjected to tensile stress.

Fig. 18 shows the variation of the peak acceleration slope for the four plates when subjected to compressive or tensile stresses. It is found that the slope is clear for a small thickness plate due to a full guided wave from the transmitter point to the receiver point. While in the rest thick plates the slopes are very small due to the distortion of the wave.



**Figure 18** Variation of peak acceleration slope of plates with different thicknesses under compressive or tensile stresses.

## 6 EFFECT OF PLATE LENGTH

In this section, three plates with different lengths are studied under the effect of different levels of compressive or tensile stresses. The wave excitation signal

is applied at the transmitter point. The results of these plates are compared with the first plate that has a dimension of 400 mm length, 200 mm width, and 2 mm thickness. Table (2) shows the plate dimensions.

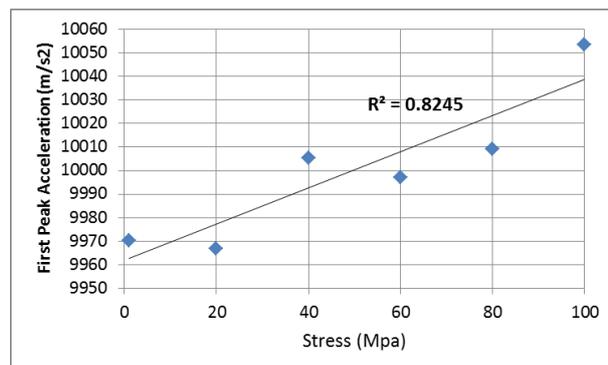
**Table 2** plates with a different length dimension

	Length (mm)	Width (mm)	Thickness (mm)
Fifth plate	500	200	2
sixth plate	600	200	2
seventh plate	700	200	2

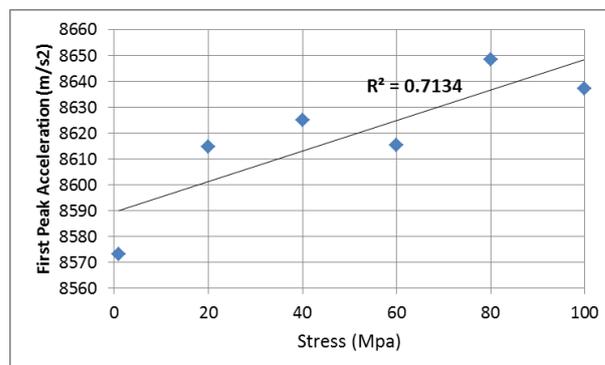
Figures 19, 20, and 21 show the relation between the first peak acceleration of the received wave and the applied stresses when the three plates are subjected to varied compressive stresses. It is shown that the peak acceleration of the received wave is slightly increased with increasing compressive stresses.

Figures 22, 23, and 24 show the relation between first peak acceleration and the applied stresses when these plates are under varied tensile stresses. It is shown that the peak acceleration of the received wave is slightly decreased with increasing tensile stresses.

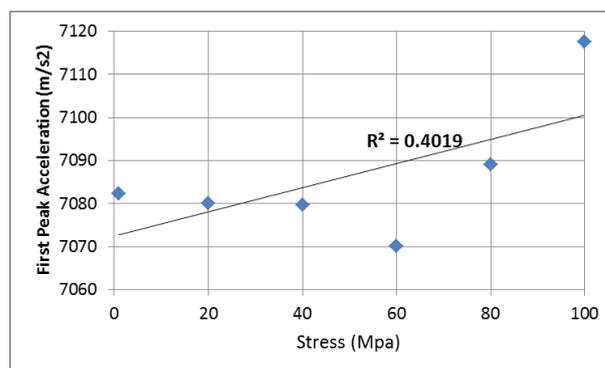
When comparing the regression factor of the six previous relations, it is found that the trend of curves becomes weak for high length plates when subjected to compressive stress due to the plate buckling.



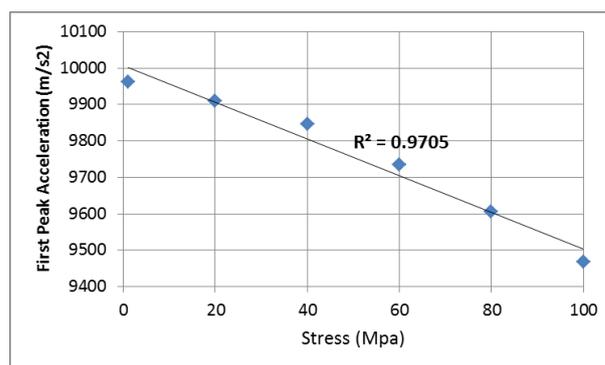
**Figure 19** Variation of peak acceleration of the fifth plate under compressive stresses.



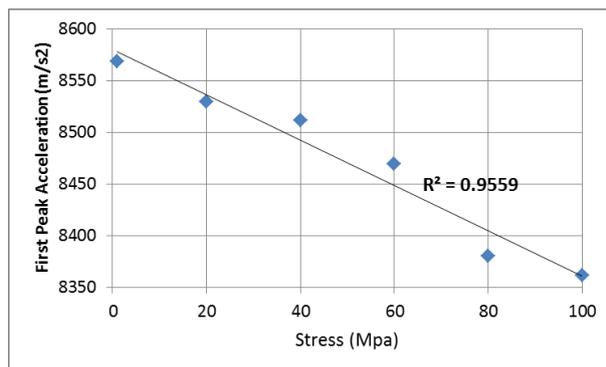
**Figure 20** Variation of peak acceleration of the sixth plate under compressive stresses.



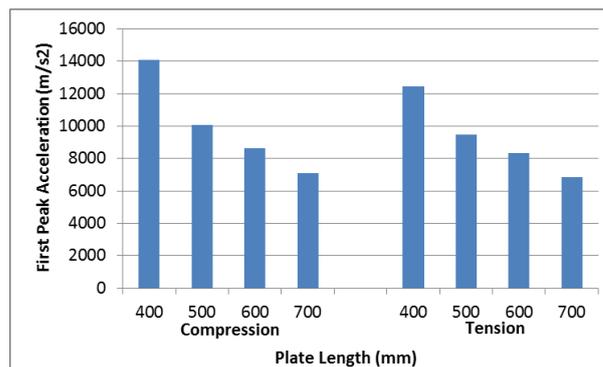
**Figure 21** Variation of peak acceleration of the seventh plate under compressive stresses.



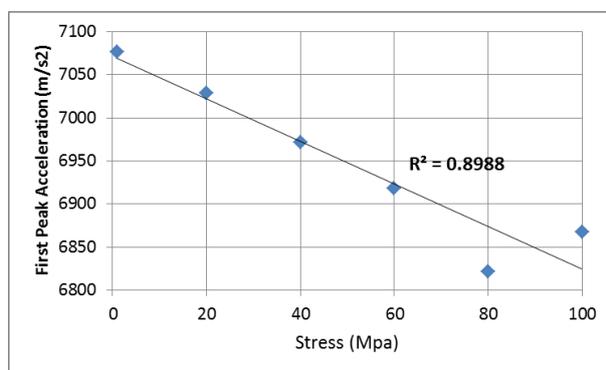
**Figure 22** Variation of peak acceleration of the fifth plate under tensile stresses.



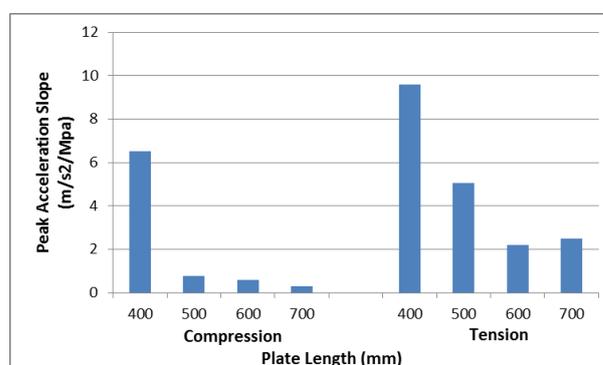
**Figure 23** Variation of peak acceleration of the sixth plate under tensile stresses.



**Figure 25** Variation of peak acceleration of plates with different lengths under compressive or tensile stress (100 Mpa).



**Figure 24** Variation of peak acceleration of the seventh plate under tensile stresses.



**Figure 26** Variation of peak acceleration slope of plates with different lengths under compressive or tensile stresses.

Fig. 25 shows the variation of the first peak acceleration for the four plates when subjected to compressive or tensile stresses 100 Mpa. It is found also that there is a slight increase in the peak acceleration of all plates when subjected to compressive stress than tensile stresses, and there is a decrease in the peak acceleration amplitude for longer plates because the wave energy is more dissipated.

Fig. 26 shows the variation of the peak acceleration slope for the four plates when subjected to compressive or tensile stresses. It is found that the slope is clear for short-length plates than longer ones, due to the full guided wave from the transmitter point to the receiver point.

## 7 EFFECT OF FREQUENCY

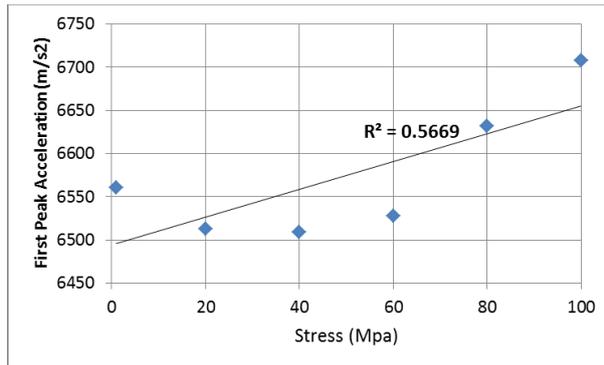
In this section the first plate is studied under different excited wave frequencies 50 kHz, 100 kHz, 250 kHz, and the plate is studied previously with a frequency of 200 kHz.

Figures 27, 28, and 29 show the relation between first peak acceleration and the applied stresses when these plates are under varied compressive stresses when excited by waves with different frequencies. It is shown that the peak acceleration of the received wave is slightly increased with increasing compressive stresses.

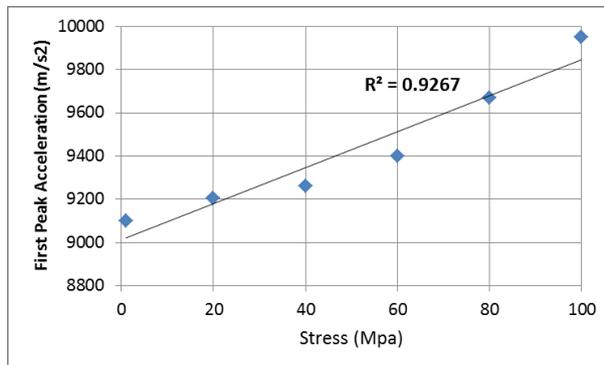
Figures 30, 31, and 32 show the relation between first peak acceleration and the applied stresses when these plates are under varied tensile stresses when excited by waves with different frequencies. It is

shown that the peak acceleration of the received wave is slightly decreased with increasing tensile stresses.

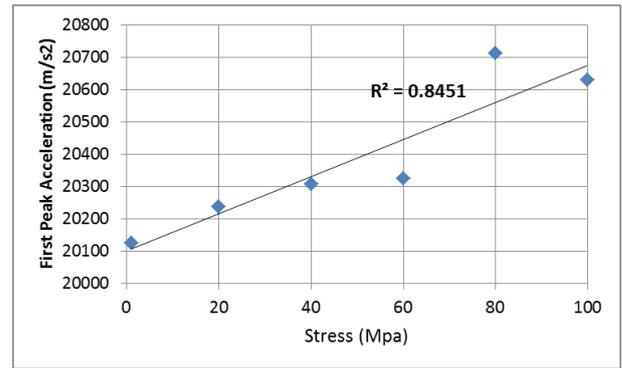
Fig. 33 shows the variation of the first peak acceleration for the plate when subjected to compressive or tensile stresses 100 Mpa with an exciting wave with different frequencies. It is found also that with increasing the frequency of exciting waves the results are clearer.



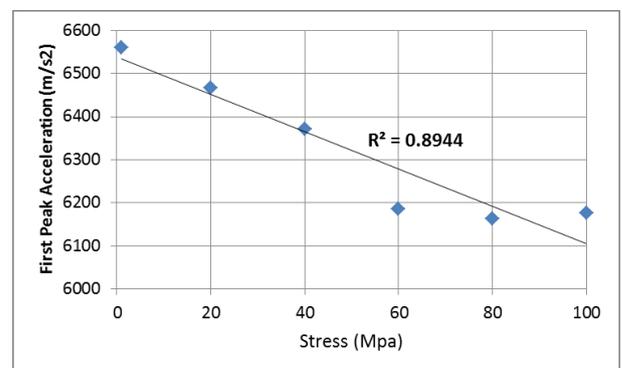
**Figure 27** Variation of peak acceleration of the first plate under compressive stresses due to wave frequency 50 kHz.



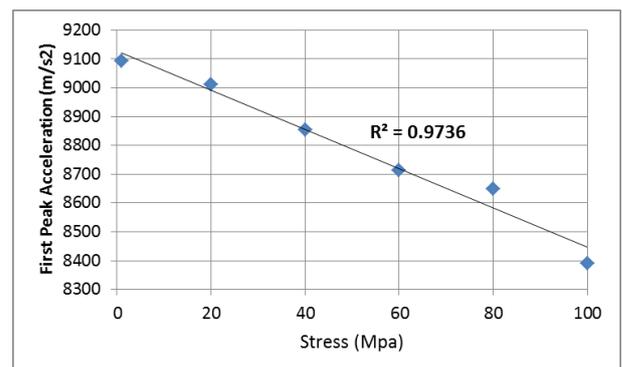
**Figure 28** Variation of peak acceleration of the first plate under compressive stresses due to wave frequency 100 kHz.



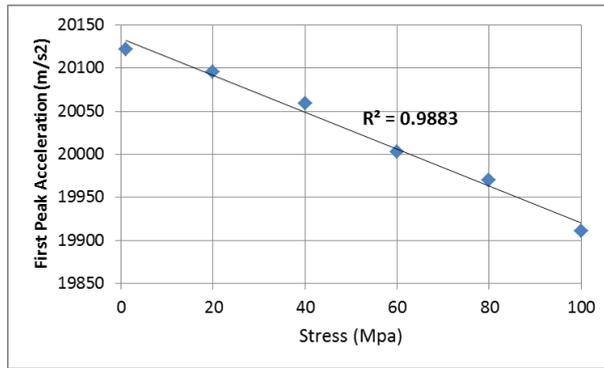
**Figure 29** Variation of peak acceleration of the first plate under compressive stresses due to wave frequency 250 kHz.



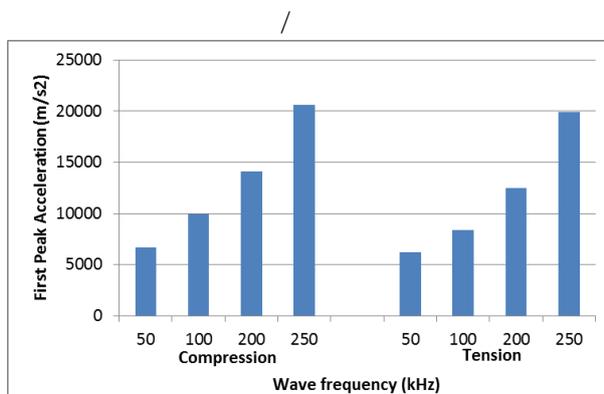
**Figure 30** Variation of peak acceleration of the first plate under tensile stresses due to wave frequency 50 kHz.



**Figure 31** Variation of peak acceleration of the first plate under tensile stresses due to wave frequency 100 kHz.



**Figure 32** Variation of peak acceleration of the first plate under tensile stresses due to wave frequency 250 kHz.



**Figure 33** Variation of peak acceleration of plates with different lengths under compressive or tensile stress (100 Mpa).

## 8 CONCLUSION

From the current analysis, the most important conclusions can be summarized as follows

In the case of compressive stress, the amplitude of the received wave increases slightly, wherein in the case of tensile stress the amplitude of the received wave decreasing slightly. With increasing plate thickness the amplitude of the received wave decrease and the effect of the stress in the wave propagation decrease where the wave loses part of its power. With increasing the path length the time of flight (TOF) of the received wave increase and its amplitude decrease. With increasing plate length the power of the received wave decreasing. With increasing frequency, the

effect of the stress in the wave propagation is clearer.

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